MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

Practice Paper C3-B

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- There is an Insert booklet for use in Question **9.**
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Prove that the product of any three consecutive integers is a multiple of 6. [4]

2 **(i)** Sketch the graph of $y = |2x - 3|$. [2]

(ii) Hence, or otherwise, solve the inequality $|2x - 3| < 5$.
Illustrate your answer on your graph. [2]

3 Differentiate the following functions.

(i) $y = \left(x^2 + 3\right)^5$ [3]

(ii) $y = \dfrac{\sin 2x}{x}$ [3]

4 A curve has equation $y^2 = 5x - 4$.

Find the gradient of the curve at the points where $x = 8$. [5]

5 Given that x and t are related by the formula $x = x_0 e^{-3t}$, show that $t = \ln\left(\dfrac{a}{x}\right)^b$ where a and b are to be determined. [4]

6 **(i)** Find $\int (2x - 3)^7 \, dx$. [3]

(ii) Use the substitution $u = x^2 + 1$, or otherwise, to find $\int_1^2 x\left(x^2 + 1\right)^3 \, dx$. [5]

7 The functions f, g and h are defined as follows.

$f(x) = 2x$ $\quad g(x) = x^2$ $\quad h(x) = x + 2$

Find each of the following as functions of x.

(i) $f^2(x)$, [1]

(ii) $fgh(x)$, [3]

(iii) $h^{-1}(x)$. [1]

Section B (36 marks)

8 A curve has equation $y = (x + 2)e^{-x}$.

(i) Find the coordinates of the points where the curve cuts the axes. [2]

(ii) Find the coordinates of the stationary point, S, on the curve. [4]

(iii) By evaluating $\frac{d^2y}{dx^2}$ at S, determine whether the stationary point is a maximum or a minimum. [3]

(iv) Sketch the curve in the domain $-3 < x < 3$. [1]

(v) Find where the normal to the curve at the point (0, 2) cuts the curve again. [3]

(vi) Find the area of the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$. [5]

9 Answer parts (i) and (iii) on the insert provided.

Fig. 9 shows a sketch graph of $y = f(x)$.

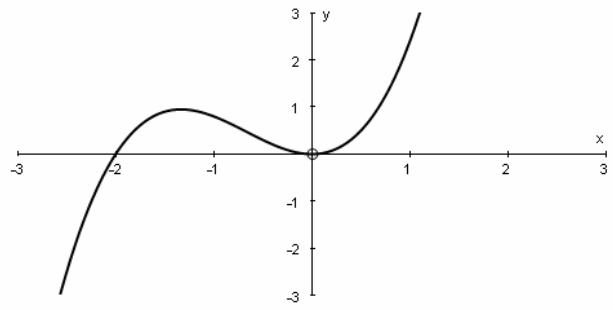


Fig. 9

(i) On the Insert sketch graphs of

(A) $y = 2f(x)$, (B) $y = f(-x)$, (C) $y = f(x - 2)$

In each case describe the transformations. [8]

(ii) Explain why the function $y = f(x)$ does not have an inverse function. [2]

(iii) The function $g(x)$ is defined as follows:

$g(x) = f(x)$ for $x \geq 0$

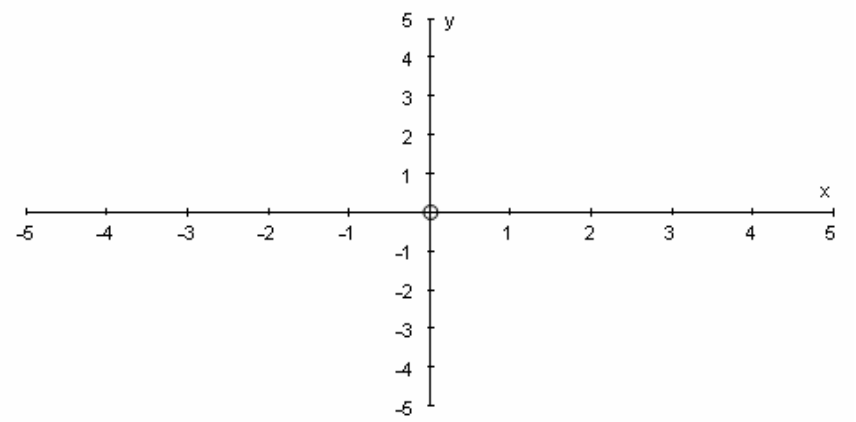
On the Insert sketch the graph of $y = g^{-1}(x)$. [1]

(iv) You are given that $f(x) = x^2(x + 2)$.
Calculate the gradient of the curve $y = f(x)$ at the point (1, 3).
Deduce the gradient of the function $g^{-1}(x)$ at the point where $x = 3$. [4]

(v) Show that $g(x)$ and $g^{-1}(x)$ cross where $x = -1 + \sqrt{2}$. [3]

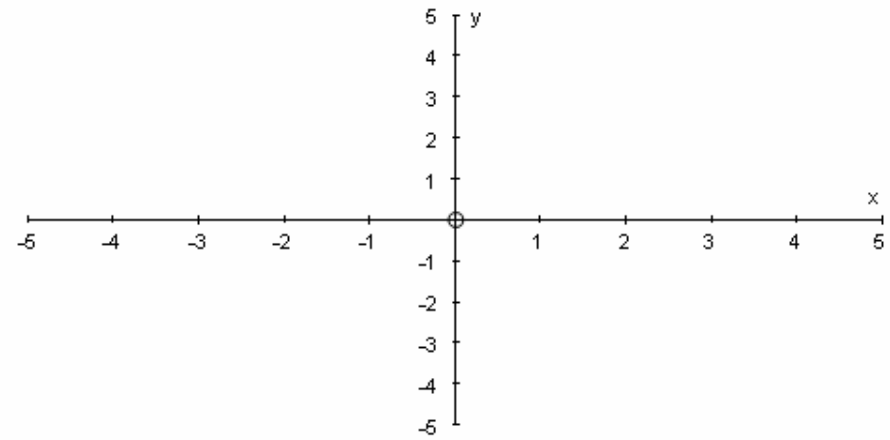
Insert for question 9.

(i) (*A*) On the axes below sketch the graph of $y = 2f(x)$.
Describe the transformation. [2]



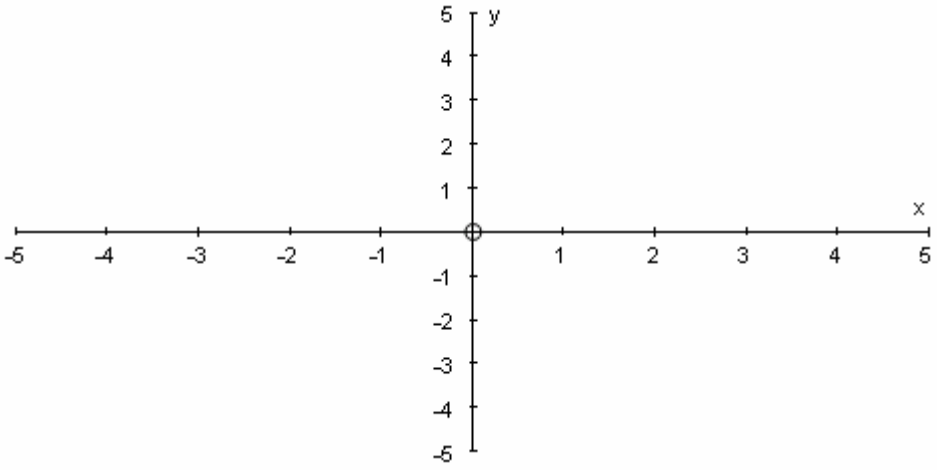
Description:

(i) (*B*) On the axes below sketch the graph of $y = f(-x)$.
Describe the transformation. [3]



Description:

(i) (*C*) On the axes below sketch the graph of $y = f(x - 2)$.
Describe the transformation. [3]



Description:

(iii) The function $g(x)$ is defined as follows:

$$g(x) = f(x) \text{ for } x \geq 0$$

On the axes below sketch the graph of $y = g^{-1}(x)$. [1]